C&PE 661: Undergraduate Honors Research

Final Presentation

Dispersion Coefficient Modeling for Unfavorable Displacement Concentration Profiles

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Friday May 4th, 2018

Outline

Objectives

Research Objective:

- The purpose for this research was to find a correlation that generates a “best fit curve” for the unfavorable displacement processes when the low concentration glycerin solutions displace the high concentration solutions at flowrates ranging from 0.61 mL/min to 20 mL/min.

Experimental Objective:

- The purpose of the Viscous Fingering in a Linear Porous Medium experiment was to examine the influence of viscous fingering on the spreading of the mixing zone in a linear displacement, estimate the pore volume from the dispersion data collected, calculate the dispersion coefficient of a miscible linear displacement, and correlate dispersion with the mobility ratio in a linear porous medium.[1]

Introduction: Viscous Fingering

- Dispersion[1]:
  - Two phases create a mixing zone which creates effect of molecular diffusion
  - Combined effect of molecular diffusion and particle level dispersion
- Viscous Fingering[2]:
  - Condition whereby the interface of two fluids bypasses sections of reservoir as it moves along, creating an uneven/fingered profile

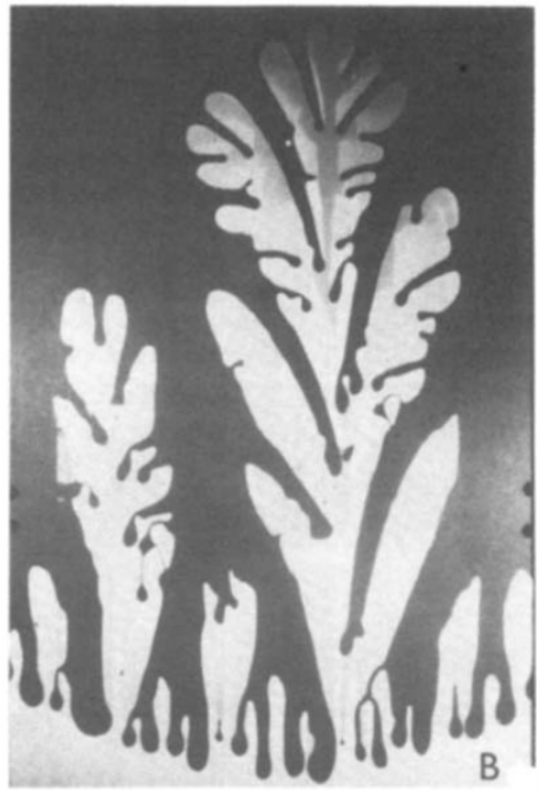


Figure 1 Viscous Fingering Patterns of a Cell in the Vertical Position [3]

General Workflow

1. Literature Review
2. Complete Viscous Fingering in a Linear Porous Medium Experiment
3. Analyze Experimental Results
4. Obtain VISF Results from other students who have completed experiment
5. Unfavorable Displacement Concentration Profile Modeling

Literature Review: Brigham[4]

- Longitudinal dispersion: $\frac{\partial C}{\partial t} = K \frac{\partial^2 C}{\partial {x_1}^2}$
- Including boundary conditions: $C = \frac{1}{2}\left[1 - erf\left(\frac{x_1}{2\sqrt{Kt}}\right)\right]$
  - "The argument $\frac{x_1}{2\sqrt{Kt}}$ indicates that at a constant rate of flow and with a constant dispersion coefficient the spread of the mixed zone will be proportional to the square root of the distance traveled."
- Relate dispersion coefficient (K) to the error function parameter U:
  - $K = \frac{1}{V_p T}\left[\frac{L(U_{90} - U_{10})}{3.625}\right]^2$

Literature Review: Perkins[5]

- Diffusion coefficients: $\frac{D}{D_o} = \frac{1}{F\phi}$
  - F is the formation electrical resistivity
  - ϕ is the porosity
- Diffusion coefficient: $D_o = \frac{1}{t}\left[\frac{X_{90} - X_{10}}{3.625}\right]^2$
- Including boundary conditions:

$$C = \frac{1}{2}\left[1 \pm \operatorname{erf}\left(\frac{0.5}{K_l/_{UL}}\right)\left(\frac{1 - {}^{V}/_{V_p}}{\sqrt{{}^{V}/_{V_p}}}\right)\right]$$

  - Where $K_l = UL\left[\frac{\lambda_{90} - \lambda_{10}}{3.625}\right]^2$

Experiment Equipment & Supplies [1]

- Linear Sandpack – contained in Glass Chromatography Column
  - Withstanding pressure of 30 psi
- Two Eldex Pumps (0-20 mL/min)
- PR 111 Refractive Index Detector
  - Output is a 4-20ma signal monitored by LabView program
  - Reichert ABBE Mark III precision refractometer used for calibration
- Reverse Osmosis (RO) water
- Glycerol (pure) – viscous liquid
- Tared Beaker for injected fluids

Experimental Setup

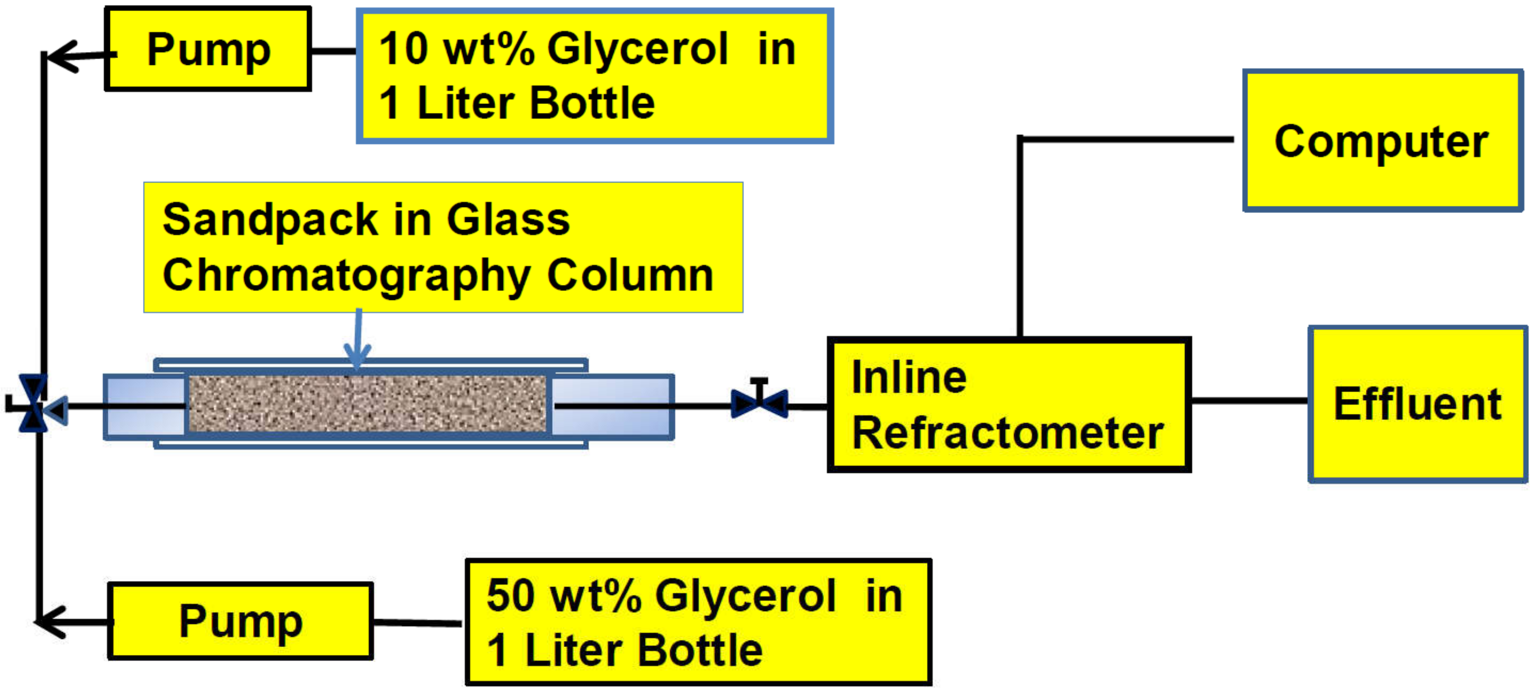


Figure 2 Schematic of Experimental Setup for Viscous Fingering in a Linear Porous Medium Experiment [1]

Experimental Procedure [1]

- Prepare nine different solutions of pure glycerol in RO water
  - Cover 10-50% by weight range
- Calibrate Equipment
  - PR 111 Refractometer
  - Eldex Pump
  - Reichert ABBE Mark III precision refractometer
    - Create a correlation of Refractive Index (RI) of Glycerin Solutions with Glycerin Concentration (wt%) at $20℃$

Experimental Procedure [1]

- Saturate porous medium with 10wt% glycerin solution
- Displacement Test: Favorable Mobility Ratio
  - Displace 10wt% glycerin solution with 50wt% glycerin solution
- Displacement Test: Unfavorable Mobility Ratio
  - Displace 50wt% glycerin solution with 10wt% glycerin solution
- Stop the run when the voltage is constant (no further change in effluent concentration)

Modeling Procedure

1. Started with provided Dispersion Coefficient and Normalized Concentration Profile Equations

2. Used equations obtained from literature review to produce a New Longitudinal Dispersion Equation

3. Defined terms and variables in the New Longitudinal Dispersion Equation using constants and ranges from literature review

4. Rewrote the New Longitudinal Dispersion Equation including the constants obtained from literature

5. Add coefficient [2] in front of front of the Sandpack Length in the numerator

6. Add coefficient [X] in front of the square root of the dispersion coefficient in the denominator

Starting Point: DLC Lab Manual [6]

Two Main Equations Derived from Brigham & Perkins:

1. Dispersion Coefficient:

$$K_l = \left[\frac{L(U_{10} - U_{90})}{3.625}\right]^2 \frac{1}{V_p t^*}$$

Nomenclature		
Symbol	Defintion	Unit
C_D	normalized concentration	unitless
K_l	dispersion coefficient	cm^2/sec
L	length of #6 – Ottawa Sand Sandpack	cm
t*	time to inject one pore volume of fluid	sec
U_{10}	value of U at 10% wt concentration	$cm^{1.5}$
U_{90}	value of U at 90% wt concentration	$cm^{1.5}$
V_i	volume of injected solution	mL
V_p	pore volume	mL

2. Normalized Concentration:

$$C_D = \frac{1}{2}\left\{1 - erf\left[\frac{1}{2\sqrt{K_l}}\left(\frac{V_p - V_i}{\sqrt{V_i}}\right)\left(\frac{L}{\sqrt{V_p t^*}}\right)\right]\right\}$$

- ❖ Using the above as the base equation for determining a correlation that generates a “best fit curve” for the unfavorable displacement concentration profiles.

Modeling

Step 1: Modeling New Longitudinal Dispersion Equation

1) $$\frac{K_l}{D_o} = \left(\frac{1}{F\phi}\right) + 0.5\left(\frac{U\sigma d_p}{D_o}\right)$$ [5]

– Where $D_o = \left(\frac{1}{t}\right)\left[\frac{X_{90}-X_{10}}{3.625}\right]^2$ [5] → Plug D_o into Eq. (1) above

2) $$\frac{K_l}{\left(\frac{1}{t}\right)\left(\frac{X_{90}-X_{10}}{3.625}\right)^2} = \left(\frac{1}{F\phi}\right) + 0.5\left[\frac{U\sigma d_p}{\left(\frac{1}{t}\right)\left(\frac{X_{90}-X_{10}}{3.625}\right)^2}\right]$$

– Solve Eq. (2) for K_l

❖ New Longitudinal Dispersion Equation:

$$K_l = \left(\frac{1}{F\phi}\right)\left[\left(\frac{1}{t}\right)\left(\frac{X_{90}-X_{10}}{3.625}\right)^2\right] + 0.5(U)(\sigma)(d_p)$$

Nomenclature		
Symbol	Defintion	Unit
D_o	molecular diffusion coefficient	cm^2/sec
d_p	particle diameter	mm
F	formation electical resistivity	unitless
K_l	dispersion coefficient	cm^2/sec
φ	porosity	fraction
σ	measure of the inhomogeneity of the pack	unitless
t	time to inject one pore volume of fluid	sec
U	flowrate	mL/min
X_{10}	value of U at 10% wt concentration	$cm^{1.5}$
X_{90}	value of U at 90% wt concentration	$cm^{1.5}$

Modeling

Step 2: Defining Terms and Variables in New Longitudinal Dispersion Equation

- $\frac{K}{D} = \frac{1}{F\phi}$
  - The term $\frac{1}{F\phi}$ commonly varies between 0.15 and 0.70.[4]
- Used Porous medium of 0.044 mm beads[4]
  - d_p = 0.044 mm
- σ is a measure of the inhomogeneity of the pack[5]
  - σ = 3.5 for a typical random pack

Modeling

Step 3: Rewrite New Longitudinal Dispersion Equation with Defined Terms and Variables

- Plug in d_p and σ values

❖ New Longitudinal Dispersion Equation:

$$K_l = \left(\frac{1}{F\phi}\right)\left[\left(\frac{1}{t}\right)\left(\frac{X_{90} - X_{10}}{3.625}\right)^2\right] + 0.5(U)(3.5)(0.044)$$

Modeling

Step 4: Modeling the Unfavorable Displacement Concentration Profile Equation

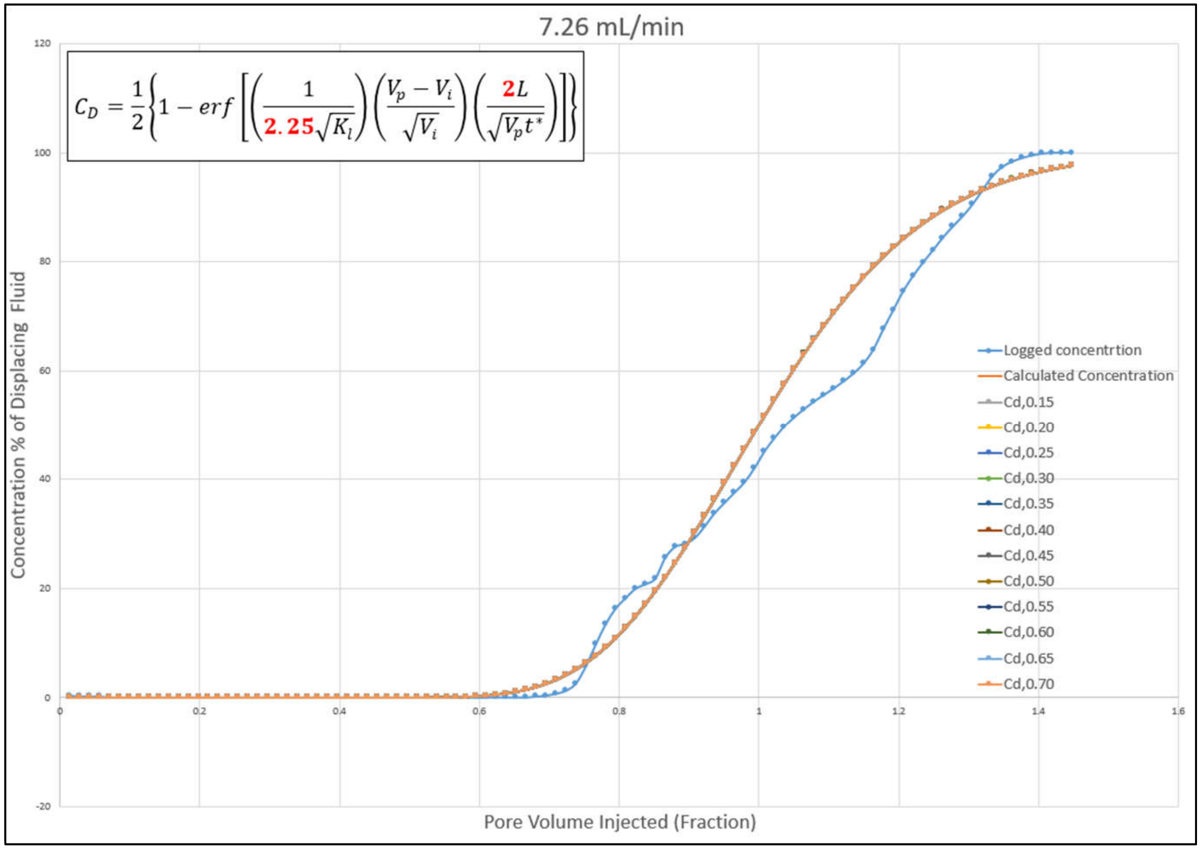
1. Beginning with the Normalized Concentration Equation[5]
   - $C_D = \frac{1}{2}\left\{1 - erf\left[\left(\frac{1}{2\sqrt{K_l}}\right)\left(\frac{V_p - V_i}{\sqrt{V_i}}\right)\left(\frac{L}{\sqrt{V_p t^*}}\right)\right]\right\}$
2. Add coefficient [2] in front of L
   - $C_D = \frac{1}{2}\left\{1 - erf\left[\left(\frac{1}{2\sqrt{K_l}}\right)\left(\frac{V_p - V_i}{\sqrt{V_i}}\right)\left(\frac{2L}{\sqrt{V_p t^*}}\right)\right]\right\}$
3. Change [2] coefficient in front of $\sqrt{K_l}$ to [X]
   - $C_D = \frac{1}{2}\left\{1 - erf\left[\left(\frac{1}{X\sqrt{K_l}}\right)\left(\frac{V_p - V_i}{\sqrt{V_i}}\right)\left(\frac{2L}{\sqrt{V_p t^*}}\right)\right]\right\}$

Results

- Outline:
  - “Clean” logged concentration data
  - “Messy” logged concentration data
  - Comparison of two similar flowrates
  - All Flowrates
  - Summary Results Table

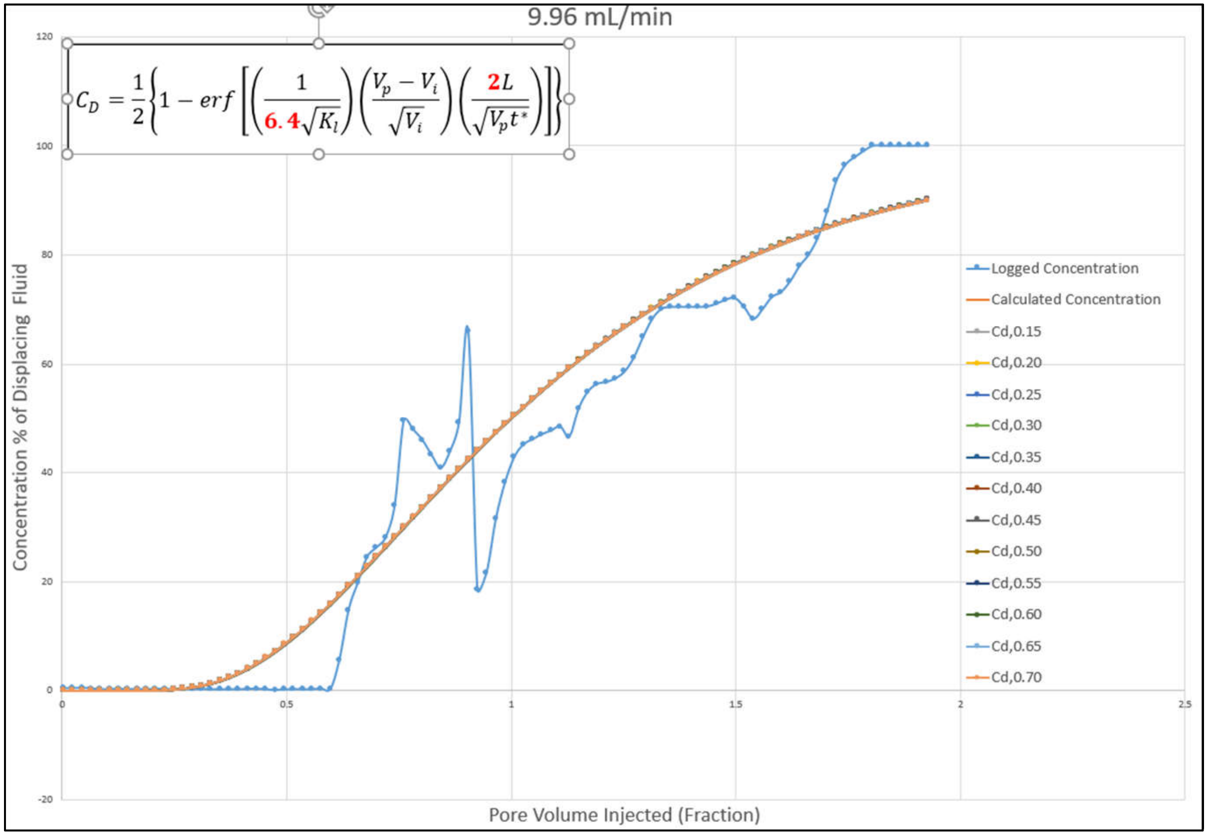
Results: “Clean” Data

Flowrate (mL/min)	Determined Coefficient
7.26	2.25



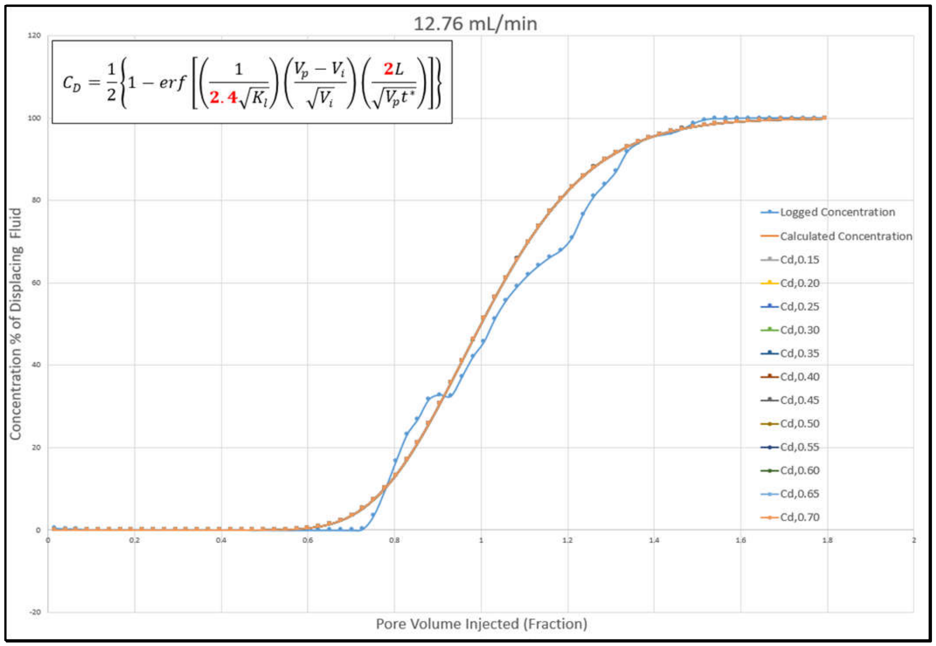
Results: “Messy” Data

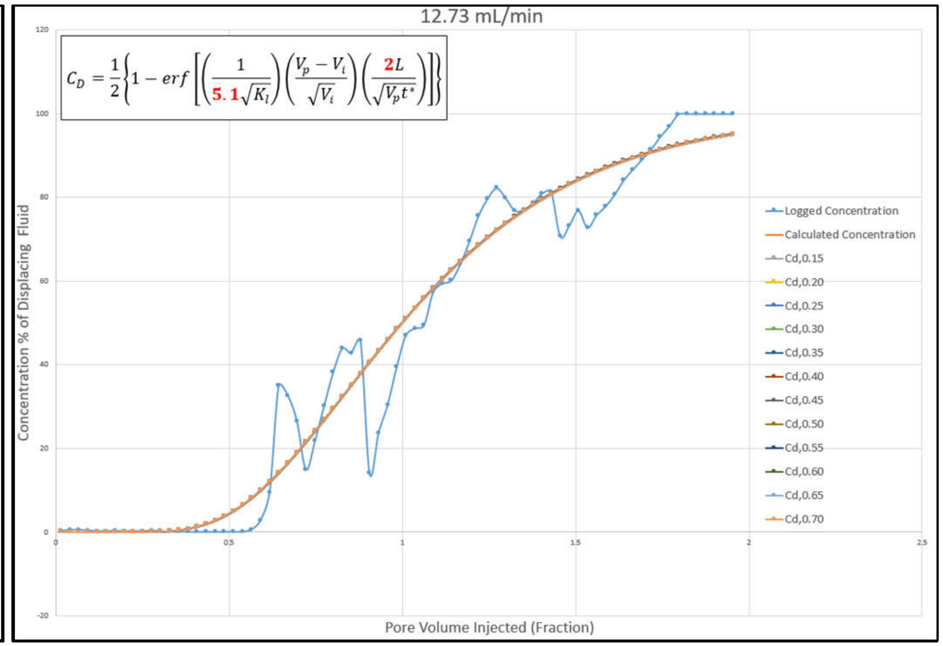
Flowrate (mL/min)	Determined Coefficient
9.96	6.4



Results: Comparison Two Similar Flowrates

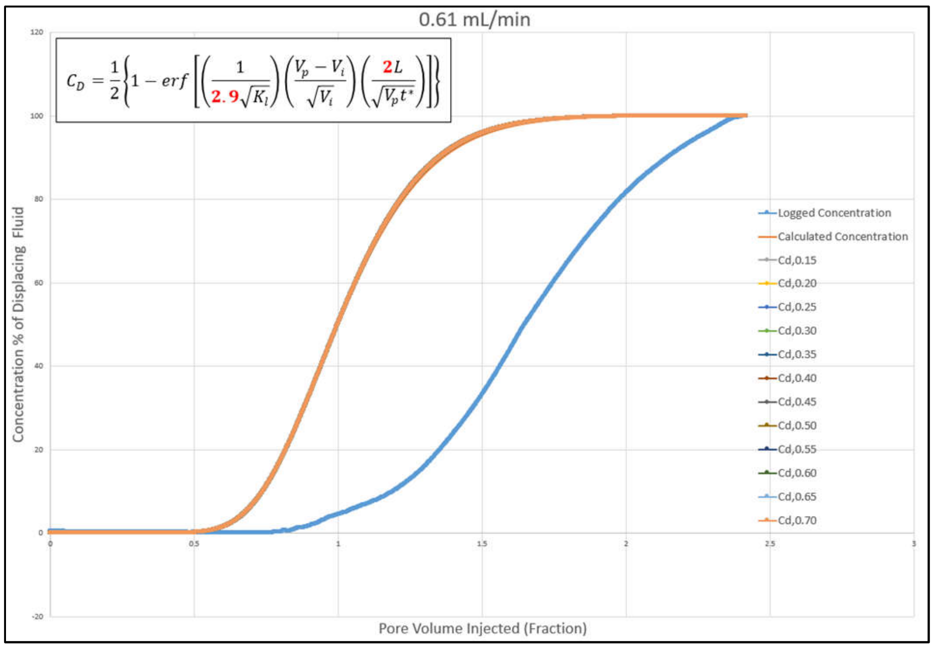
Flowrate (mL/min)	Determined Coefficient
12.76 → “clean” data	2.4
12.73 → “messy” data	5.1

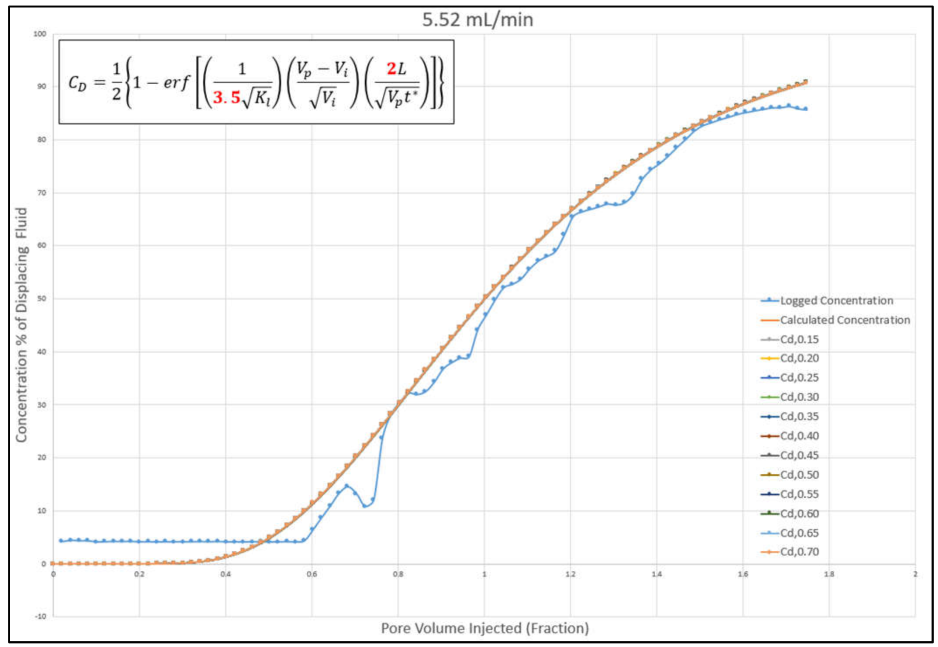




Results – All Flowrates

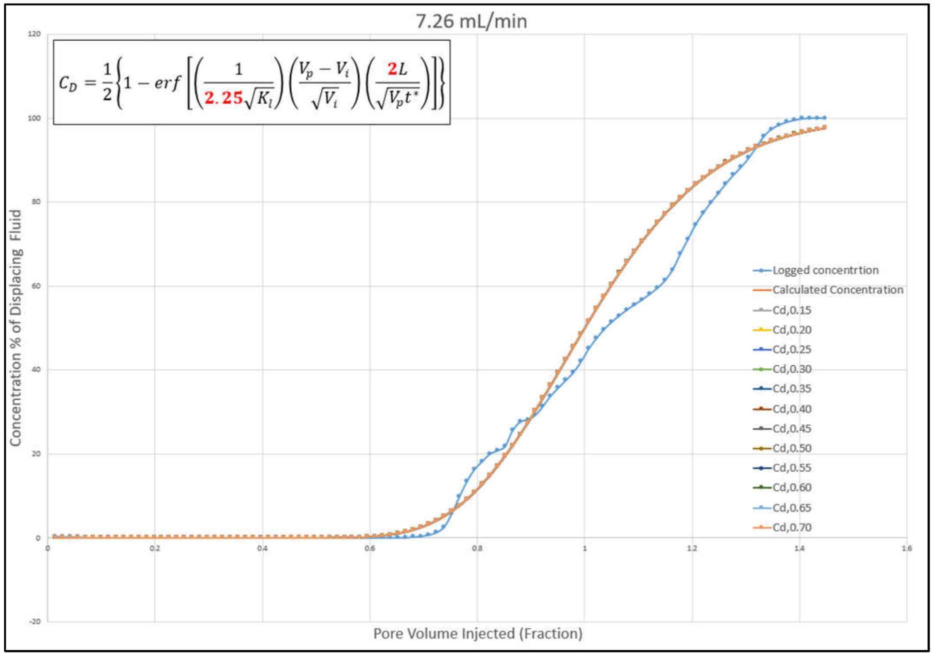
Flowrate (mL/min)	Determined Coefficient
0.61	2.9
5.52	3.5

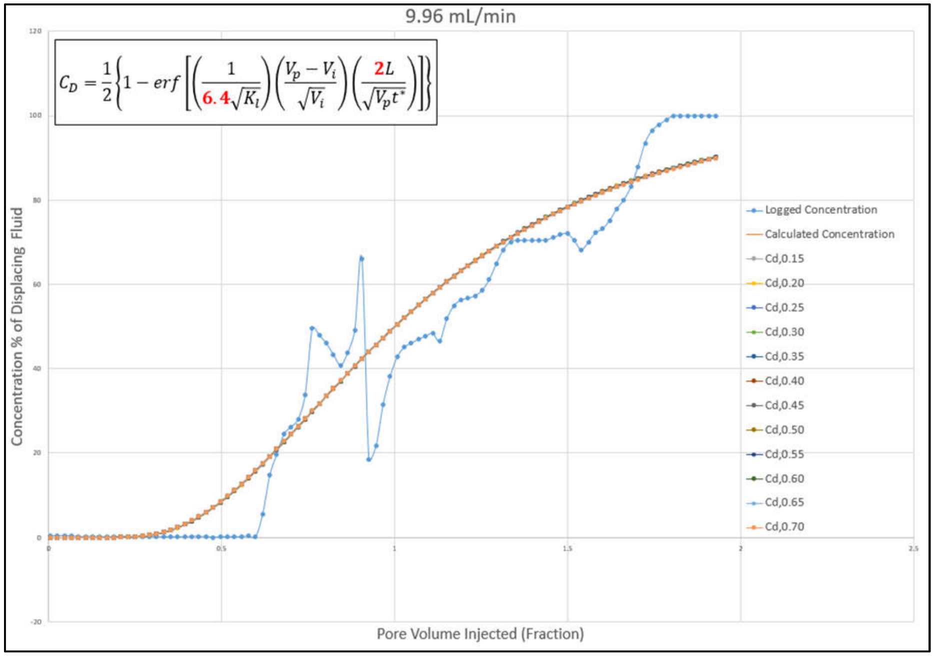




Results – All Flowrates

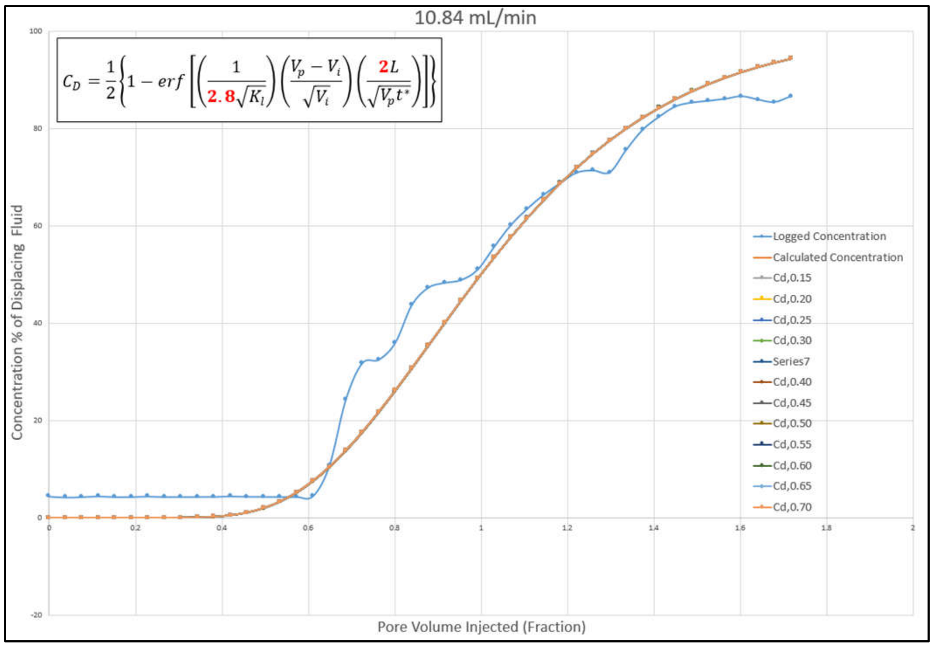
Flowrate (mL/min)	Determined Coefficient
7.26	2.25
9.96	6.4

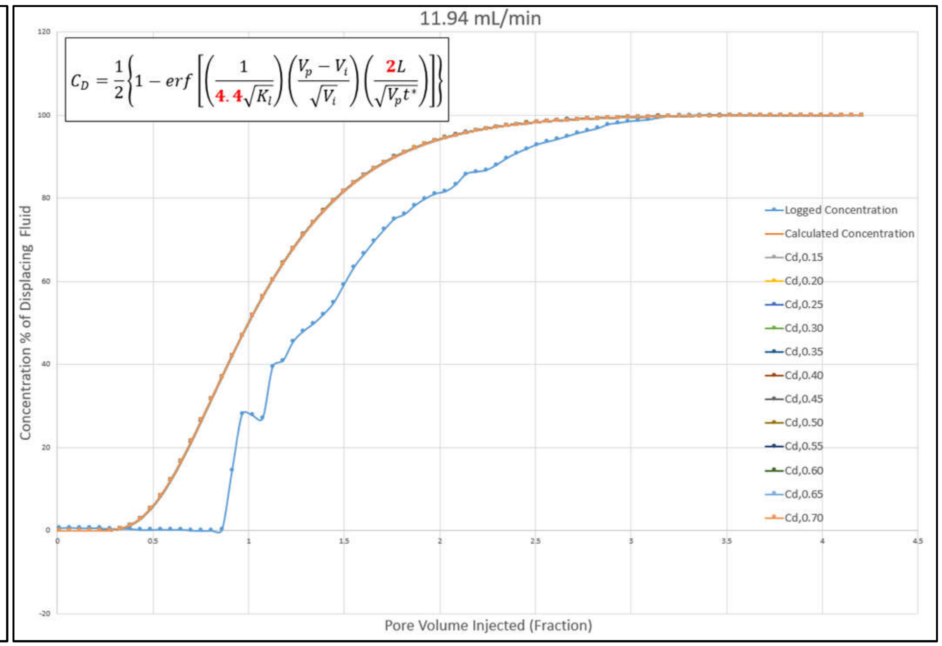




Results – All Flowrates

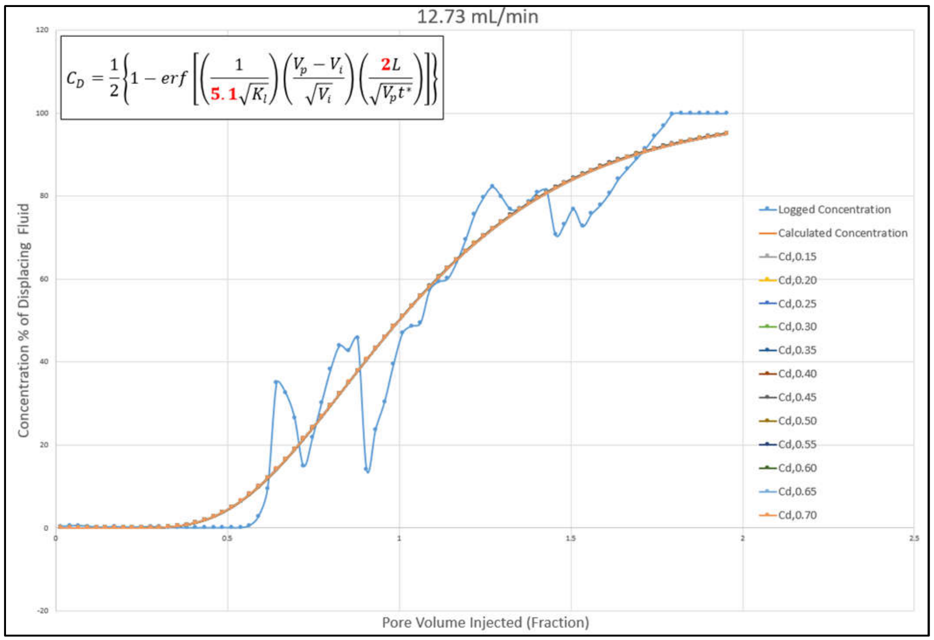
Flowrate (mL/min)	Determined Coefficient
10.84	2.8
11.94	4.4

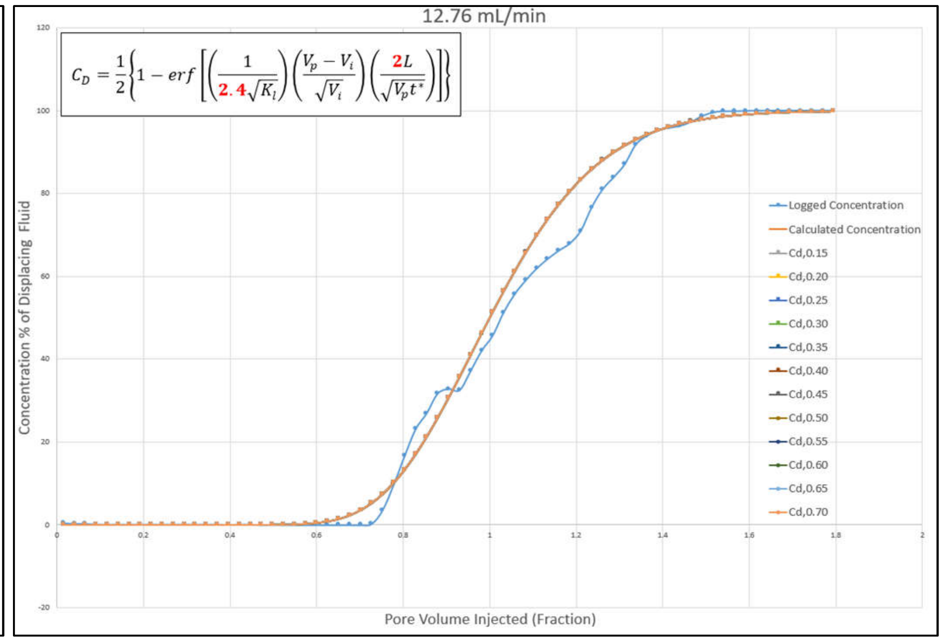




Results – All Flowrates

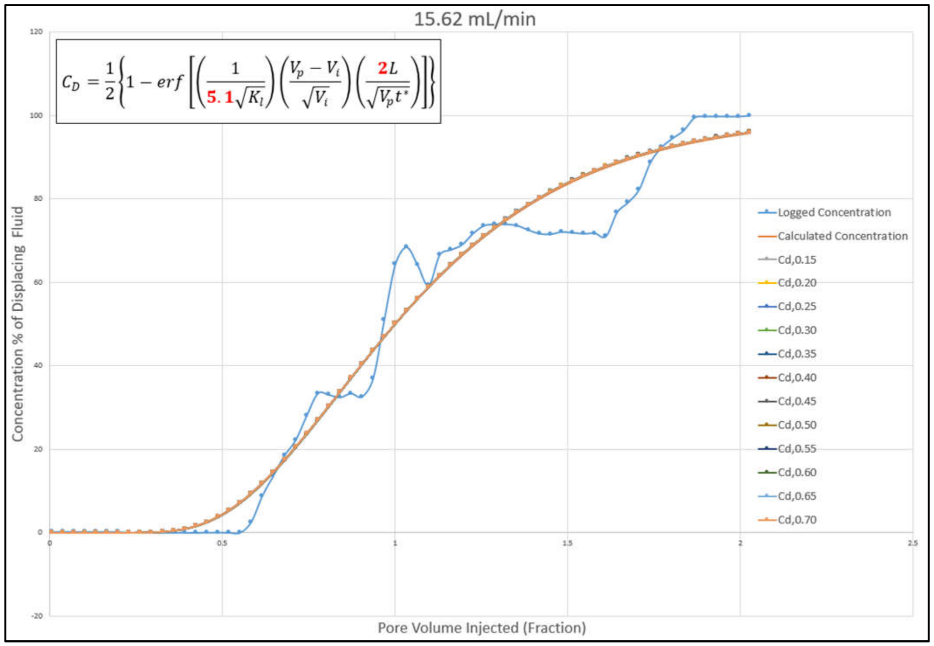
Flowrate (mL/min)	Determined Coefficient
12.73	5.1
12.76	2.4

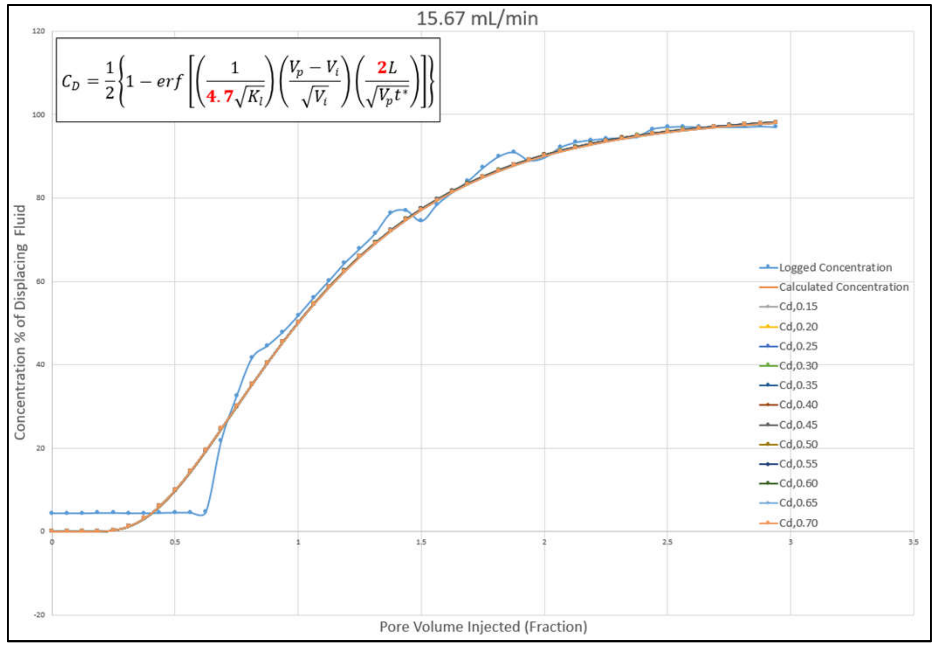




Results – All Flowrates

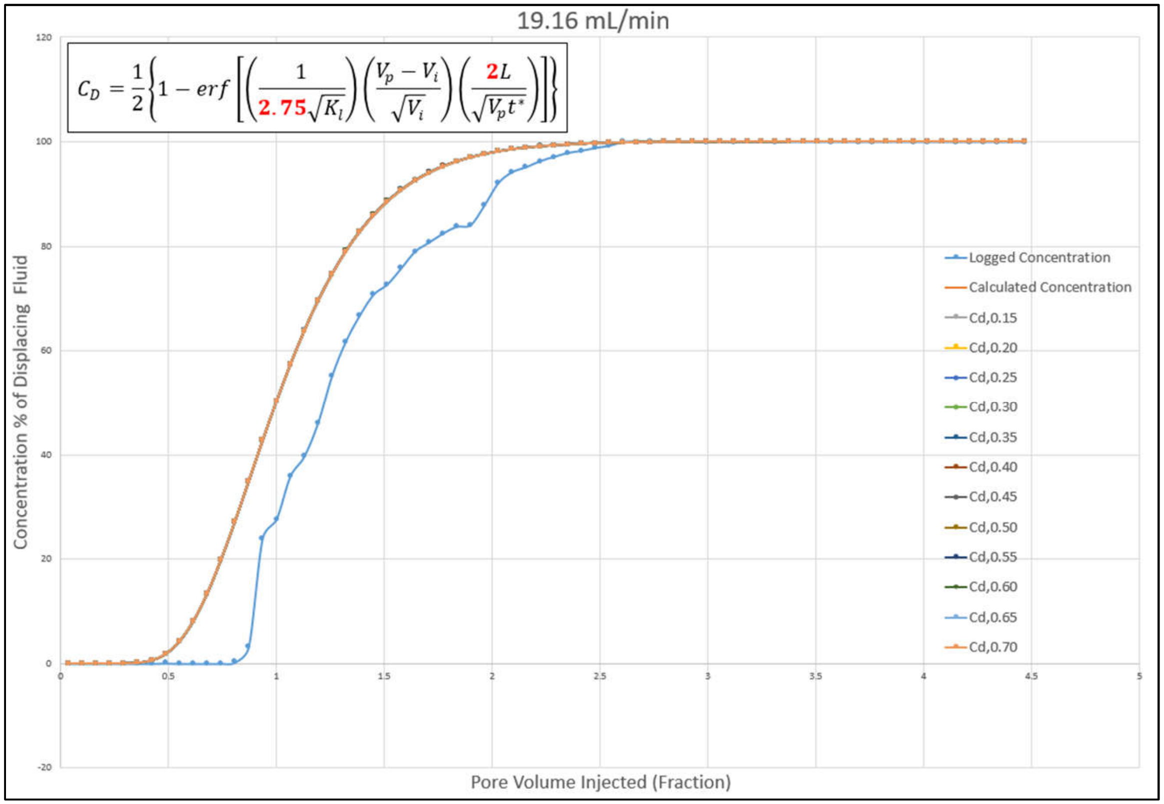
Flowrate (mL/min)	Determined Coefficient
15.62	5.1
15.67	4.7





Results – All Flowrates

Flowrate (mL/min)	Determined Coefficient
19.16	2.75



Final Results

Flowrate(mL/min)	Determined Coefficient	Error Percentage
0.61	2.9	57.93%
5.52	3.5	28.69%
7.26	2.25	26.95%
9.96	6.4	31.59%
10.84	2.8	28.54%
11.94	4.4	22.91%
12.73	5.1	29.23%
12.76	2.4	18.34%
15.62	5.1	32.33%
15.67	4.7	13.69%
19.16	2.75	21.97%

Conclusions

- Addition of coefficient [**2**] in front of the Sandpack Length in the numerator

$$C_D = \frac{1}{2}\left\{1 - erf\left[\left(\frac{1}{2\sqrt{K_l}}\right)\left(\frac{V_p - V_i}{\sqrt{V_i}}\right)\left(\frac{\mathbf{2}L}{\sqrt{V_p t^*}}\right)\right]\right\}$$

  – This constant helps the unfavorable displacement concentration profiles take the orientation of the calculated concentration profile.

Conclusions

- Addition of coefficient [**X**] in front of the square root of the dispersion coefficient in the denominator

$$C_D = \frac{1}{2}\left\{1 - erf\left[\left(\frac{1}{\boldsymbol{X}\sqrt{K_l}}\right)\left(\frac{V_p - V_i}{\sqrt{V_i}}\right)\left(\frac{2L}{\sqrt{V_p t^*}}\right)\right]\right\}$$

  - This coefficient was added to the above equation in order to match the calculated concentration profile at varying flowrates.
  - At this point, a range of determined coefficients can be recommended. However, this range applies ***ONLY*** to the observed data set.

Data Set Flowrate Range (mL/min)	Recommended Range of Determined Coefficient
0.61 – 19.16	2.25 – 6.4

Challenges

- Completing the VISF experiment only one time this semester
  - Obtaining quality data for modeling
    - Needed data for several flowrates ranging from 0 mL/min to 20 mL/min.
    - Needed data for similar (or same) flowrates for comparison of the determined coefficient.
  - Pumps are not consistently pumping through test duration time.

Recommendations for Future Work

- Researchers should perform the Viscous Fingering in a Linear Porous Medium experiment at several different flowrates ranging from 0 mL/min to 20 mL/min.
  - Several trials should be completed for each tested flowrate so that the logged data and calculated coefficient may be compared appropriately.
- Researchers should have the nine glycerin solutions prepared prior to arriving to the laboratory in order to save time on the calibration process such that more time can be spent on the displacement trials.

Acknowledgements

Dr. Li

References

1. Willhite, G. Paul. “Experiment VISF-Viscous Fingering in a Linear Porous Medium.” Lab Handbook. University of Kansas. Lawrence. 3 April 2016. Print.
2. Schlumberger. “Viscous Fingering.” Oilfield Glossary. http://www.glossary.oilfield.slb.com/en/Terms/m/miscible_displacement.aspx. 5 February 2017.
3. Homsy, G.M. “Viscous Fingering in a Porous Media.” Annual Review of Fluid Mechanics. 1987. Vol. 19: (271-311). Print.
4. Brigham, W. E., Reed, P. W., & Dew, J. N. (1961, March 1). Experiments on Mixing During Miscible Displacement in Porous Media. Society of Petroleum Engineers. doi:10.2118/1430-G
5. Perkins, T. K., & Johnston, O. C. (1963, March 1). A Review of Diffusion and Dispersion in Porous Media. Society of Petroleum Engineers. doi:10.2118/480-PA
6. Willhite, G. Paul. “Experiment DLC Dispersion in a Linear Porous Medium.” Lab Handbook. University of Kansas. Lawrence. 6 March 2017. Print.

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Questions

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